Dynamic Insider Trading and Return Predictability

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Abstract

We develop a dynamic insider trading model of markets where informed participation and price impact are jointly determined. In a CARA Normal rational expectations environment, equilibrium prices are linear in private and public signals as well as noise-trader order flow. We derive closed form expressions for adjacent horizon return covariances, showing that short-horizon predictability follows a quadratic "Lambda Law" in market impact - thin markets (high λ) generate strong reversals, while sufficiently strong learning regimes can produce short-run momentum. We extend the framework by endogenizing insider entry. The certainty-equivalent gain from private information is strictly decreasing in insider share, yielding a unique free-entry fixed point where informational rents are competed away. This "Law of Insider Motion" formalizes the feedback between profitability, entry, and market depth: shocks to noise-trader variance or information cost displace insider mass, but participation dynamics restore stability. Our model links microstructure features to return autocovariances and highlights conditions under which momentum, reversal, and fragility arise endogenously.

Keywords: Insider trading; return predictability; Lambda Law; dynamic entry.

JEL: G12, G14, C61, D83.

1. Introduction

Briefly explain the motivation, literature, and main contribution.

2. Model

2.1. A simple two-period model

In the spirit of the rational expectations general equilibrium models of Hellwig(1980) and Grossman and Stiglitz (1980), we consider a setup with with time periods $t \in \{1, 2, 3\}$ with liquidation value and terminal payoff at $t = 4$ given by $V = \theta \sim \mathcal{N}(0, \sigma_\theta^2)$. [1] The risk-free rate is normalized to 0 and the unconditional mean of θ is normalized to 0 (see footnote on normalization). A fraction $\mu \in (0, 1)$ of traders are informed, $1 - \mu$ are uninformed, and noise traders submit exogenous order $z_t \sim \mathcal{N}(0, \sigma_z^2)$ at $t = 2$. Traders are CARA with coefficient $a > 0$ and maximize $U(W) = -\exp\{-aW\}$.At time $t = 1$, there is a public signal $p = \theta + \zeta$ observed by all market participants and at time $t = 2$, a priate signal $s = \theta + \tau$ observed only by the insider with (θ, ζ, τ) jointly independent Gaussian, mean zero, and variances $\mathbb{V}\text{ar}(\theta) = \sigma_\theta^2$, $\mathbb{V}\text{ar}(\zeta) = \sigma_p^2$, $\mathbb{V}\text{ar}(\tau) = \sigma_s^{2}$[2]. Let the corresponding precisions be $\phi_\theta = \sigma_\theta^{-2}$, $\phi_p = \sigma_p^{-2}$, $\phi_s = \sigma_s^{-2}$. The equilibrium price at time $t = 3$, is trivial, i.e. $P_3 = \theta$.For all $t < 3$, we solve for the equilibrium prices through backward induction. Agents have CARA preferences and maximize end period utility over wealth $U_{i,t} = \exp -AW_{i,t}$. Hence, the corresponding CARA-normal demands of the agents can be given as

$$X_{U,2} = \frac{\mathbb{E}[\theta \mid p] - P_2}{a\,\mathbb{V}\text{ar}(\theta \mid p)}, \tag{1}$$

$$X_{I,2} = \frac{\mathbb{E}[\theta \mid s] - P_2}{a\,\mathbb{V}\text{ar}(\theta \mid s)}. \tag{2}$$

[1] Without loss of generality we set the unconditional mean of θ to 0 (and the risk-free rate to 0). In a CARANormal setting with Gaussian signals, any nonzero mean $m_\theta = \mathbb{E}[\theta]$ can be absorbed by demeaning $\tilde{\theta} = \theta - m_\theta$, $\tilde{p} = p - m_\theta$, and $\tilde{s} = s - m_\theta$. All our results hold promise even if this assumption is relaxed

[2] We work on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t=0}^4, \mathbb{P})$ supporting a jointly Gaussian vector $(\theta, \zeta, \tau, z_2) \sim \mathcal{N}(0, \Sigma)$ with $\theta \perp\!\!\!\perp \zeta \perp\!\!\!\perp \tau \perp\!\!\!\perp z_2$, $\mathbb{V}\text{ar}(\theta) = \sigma_\theta^2$, $\mathbb{V}\text{ar}(\zeta) = \sigma_p^2$, $\mathbb{V}\text{ar}(\tau) = \sigma_s^2$, $\mathbb{V}\text{ar}(z_2) = \sigma_z^2$.

Results from the projection theorem give us

$$\mathbb{E}[\theta \mid p] = \frac{\phi_p}{\phi_\theta + \phi_p}\, p, \qquad \mathbb{V}\mathrm{ar}(\theta \mid p) = \frac{1}{\phi_\theta + \phi_p}, \tag{3}$$

$$\mathbb{E}[\theta \mid s] = \frac{\phi_s}{\phi_\theta + \phi_s}\, s \qquad \mathbb{V}\mathrm{ar}(\theta \mid s) = \frac{1}{\phi_\theta + \phi_s}. \tag{4}$$

Market clearing and pricing rule:. Market clear when agents' demands are met, giving us the clearing rule

$$\mu X_{I,2} + (1-\mu) X_{U,2} + z_2 = 0 \tag{5}$$

We conjecture a linear price $P_2 = \alpha_2 s + \beta_2 p + \gamma_2 z_2$. Solving for the clearing conditon yields[3]

$$P_2 \;=\; \alpha_2\, s \;+\; \beta_2\, p \;+\; \gamma_2\, z_2 \qquad \text{with} \qquad \begin{cases} \alpha_2 = \dfrac{\mu\,\phi_s}{\Phi_2}, \\[2ex] \beta_2 = \dfrac{(1-\mu)\,\phi_p}{\Phi_2}, \\[2ex] \gamma_2 = \dfrac{a}{\Phi_2}, \end{cases} \tag{6}$$

$$\Phi_2 \equiv \phi_\theta + \mu\,\phi_s + (1-\mu)\,\phi_p. \tag{7}$$

Assumption 2.1 (CARANormal, competitive, exogenous information)**.** (i) Agents have CARA utility $U_t(w) = -\exp\{-a_t w\}$ with $a_t > 0$; (ii) $(\theta, \{\zeta_{t^-}\}, \{\tau_t\}, \{z_t\})$ are jointly Gaussian with the standard independence restrictions stated in the main text; (iii) Agents are price takers and the information flow is exogenous (today's position does not affect the law of future signals or prices other than through wealth).

Equilibrium at $t = 1$.. At $t = 1$ both informed and uninformed observe only the public signal $p = \theta + \zeta$. Let $z_1 \sim \mathcal{N}(0, \sigma_z^2)$ be the noise order at $t = 1$. With symmetric information, the aggregate demand of both class of traders is given as

$$X_{I,1} + X_{U,1} = \frac{\mathbb{E}[\theta \mid p] - P_1}{a\,\mathbb{V}\mathrm{ar}(\theta \mid p)}. \tag{8}$$

Hence the linear pricing rule[4]

[3]If $\mathbb{E}[\theta] = \bar{\theta} \neq 0$, add an intercept $\frac{\phi_\theta}{\phi_\theta + \mu\phi_s + (1-\mu)\phi_p}\,\bar{\theta}$ to (7).

[4]If $\mathbb{E}[\theta] = \bar{\theta} \neq 0$, add $\frac{\phi_\theta}{\phi_\theta + \phi_p}\,\bar{\theta}$ to P_1.

$$P_1 \;=\; \beta_1\, p \;+\; \gamma_1\, z_1 \qquad \text{with} \qquad \begin{cases} \beta_1 = \dfrac{\phi_p}{\Phi_1}, \\[2ex] \gamma_1 = \dfrac{a}{\Phi_1}, \end{cases} \tag{9}$$

$$\Phi_1 \equiv \phi_\theta + \phi_p. \tag{10}$$

2.2. Short- and Intermediate-horizon Covariances

Throughout this subsection we use the pricing rules in (7) and (10), the signal definitions $s = \theta + \tau$ and $p = \theta + \zeta$, and the mutual independence and mean-zero assumptions for $(\theta, \zeta, \tau, z_1, z_2)$. We also assume $\mathbb{V}\mathrm{ar}(z_1) = \mathbb{V}\mathrm{ar}(z_2) = \sigma_z^2$ and $z_1 \perp\!\!\!\perp z_2$. We now define the one-period returns $R_1 \equiv P_2 - P_1$ and $R_2 \equiv P_3 - P_2 = \theta - P_2$, and the two-period intermediate horizon return $R_{1,3} \equiv P_3 - P_1 = \theta - P_1$.

Proposition 2.2 (Short and intermediate-horizon return covariances)**.** *Under the information structure $s = \theta + \tau$, $p = \theta + \zeta$, with mutually independent, mean-zero $(\theta, \tau, \zeta, z_1, z_2)$ and variances $\mathbb{V}\mathrm{ar}(\theta) = \sigma_\theta^2$, $\mathbb{V}\mathrm{ar}(\tau) = \sigma_s^2$, $\mathbb{V}\mathrm{ar}(\zeta) = \sigma_p^2$, $\mathbb{V}\mathrm{ar}(z_t) = \sigma_z^2$, let precisions be $\phi_\theta = \sigma_\theta^{-2}$, $\phi_s = \sigma_s^{-2}$, $\phi_p = \sigma_p^{-2}$. With $P_2 = \alpha_2 s + \beta_2 p + \gamma_2 z_2$ and $P_1 = \beta_1 p + \gamma_1 z_1$ the one-lag (short-horizon) and short-vs.-two-period (intermediate-horizon) covariances are given to be* [5]

$$\mathcal{S} = \mathbb{C}\mathrm{ov}(R_1, R_2) = \underbrace{\frac{\mu(1-\mu)\big(\phi_s + \phi_p\big)}{\Phi_2^2} \;-\; \frac{\mu\,\phi_p}{\Phi_1\,\Phi_2}}_{\textit{fundamental (information)}} \;+\; \underbrace{\left(-\,\frac{a^2}{\Phi_2^2}\,\sigma_z^2\right)}_{\textit{inventory (order flow)}} \tag{11}$$

$$\mathcal{L} = \mathbb{C}\mathrm{ov}(R_1, R_{1,3}) = \underbrace{\frac{\mu\,\phi_s}{\Phi_1\,\Phi_2}}_{\textit{fundamental (information)}} \;+\; \underbrace{\frac{a^2}{\Phi_1^2}\,\sigma_z^2}_{\textit{inventory (order flow)}} \;. \tag{12}$$

Information is an industry with free (or at least elastic in the real sense) entry. When short-horizon trading profits attributable to private signals are abundant, more capital pays the fixed and variable costs of becoming informed , i.e. buying data, talent, and technology which is exactly the competition mechanism emphasized by **?**]. As informed participation expands, prices incorporate the private signal sooner and more completely; market depth increases and price impact falls, so the incremental value of information is competed away see [**? ?**]. This crowding logic delivers a natural negative feedback. Profits naturally attract entry; entry raises informational depth; depth

[5] For proof, see

compresses profits. We have reason to belive that this same mechanism organizes short-horizon return patterns. With few informed traders, new private information diffuses slowly and produces return continuation a.k.a momentum as prices learn across periods. As informed participation grows, two forces push back, namely (i) public-signal repricing meaning later trades re-weight away from yesterdays public signal toward todays private signal. (ii) Inventory pressure that is later unwound. We show that together they generate a hump in short-run predictability: momentum at intermediate informed share; mean reversion when informed trading is heavy, consistent with asymmetric-information microstructure accounts of return autocorrelation [e.g., **?**]nd with the broader link between liquidity and expected returns as documented by [**?**]. Especially in stressed states, when funding constraints bind or noise-trader activity is elevated, temporary impact rises and short-horizon reversals strengthen with the classic funding-liquidity spirals and loss spirals as shown by **?**] and the momentum crash narrative in **?**].
Real-world episodes often fit this loop. Alternative data waves and newer advanced analytics create an early adopter edge; rapid diffusion across funds compresses that edge as prices internalize the signal (with depth up and correspondingly impact down). Around scheduled disclosures (for example earnings, macro prints), order flow is more aligned with fundamentals, so short-run predictability temporarily rises before being competed away by subsequent entry and deeper markets (cite papers). Regulatory and technology shocks for e.g., research unbundling, market-structure changes, and algorithmic adoption shift costs, depth, and impact in the directions as documented by **?** **?** **?**]. Empirically, informed participation can be proxied by the Probability of Informed Trading (PIN) and related measures [**?** **?**]. Price impact can be estimated by intraday Kyle/Hasbrouck estimates or by the method used by Sadka(2006) originally incorporating the fundamental determinants of the bid-ask spread as proposed by Glosten and Harris(1988).

3. Dynamic Model

For our model, we consider a T-period version of the previous section with dynamic insider entry. Time is discrete, $t = 1, 2, \ldots T$, with terminal payoff $P_T = \theta$. Agents are competitive CARA utility maximizers with period t risk aversion $a_t > 0$. At each trading

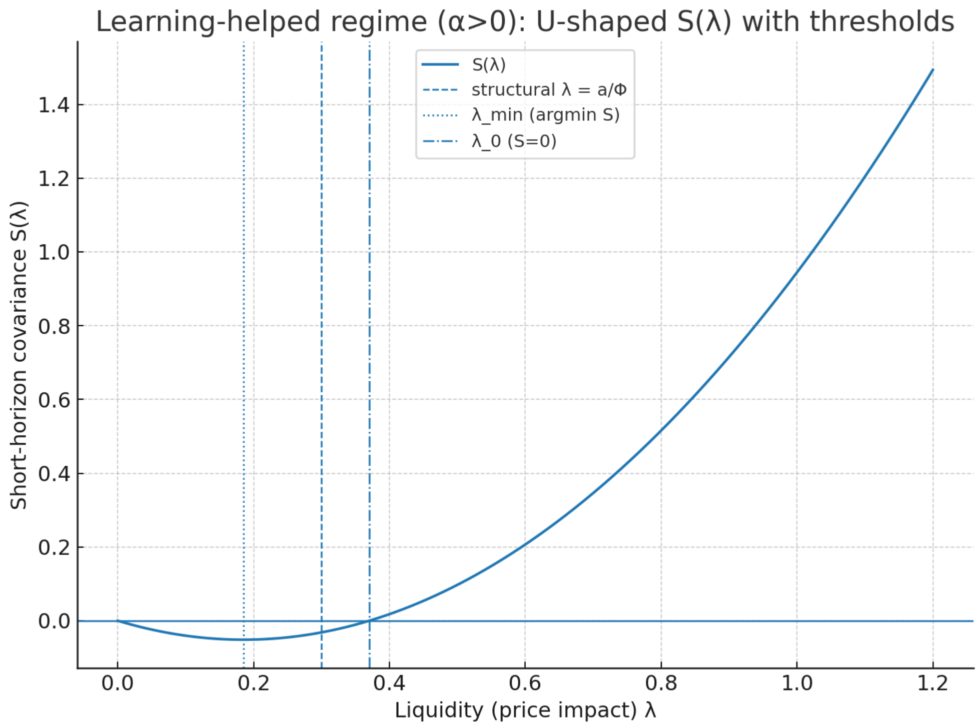

Figure 1: Learning-helped regime ($\alpha_t > 0$). The short-horizon covariance $S(\lambda)$ is quadratic in λ with a unique minimum at $\lambda_{\min} = \beta_t/(2\alpha_t)$ and a sign flip at $\lambda_0 = \beta_t/\alpha_t$. The dashed line marks the structural impact $\lambda = a/\Phi$, which determines the realized regime.

date $t < 5$ (a) A public signal $p_{t^-} = \theta + \zeta_{t^-}$ is observed by all, where "t^-" denotes the most recent public update prior to trade t. (b) A fraction $\mu_t \in [0, 1]$ of traders observe a private signal $s_t = \theta + \tau_t$. (c) Non-informational order flow or noise is $z_t \sim \mathcal{N}(0, \sigma^2_{z,t})$, independent across t and of $(\theta, \zeta_{t^-}, \tau_t)$. Let the prior be $\theta \sim \mathcal{N}(\bar{\theta}, \sigma^2_\theta)$ with precision $\phi_\theta = \sigma_\theta^{-2}$. We denote signal precisions by $\phi_{p,t^-} = \mathbb{V}\mathrm{ar}(\zeta_{t^-})^{-1}$ and $\phi_{s,t} = \mathbb{V}\mathrm{ar}(\tau_t)^{-1}$. Now, define cumulative (posterior) precision mass at t

$$\Phi_t \;\equiv\; \phi_\theta \;+\; \sum_{k\le t} \phi_{p,k^-} \;+\; \sum_{k\le t} \mu_k\, \phi_{s,k}. \tag{13}$$

In line with Luo, Subramanyam and Titman (2022) who use the loadings on the noise trader component as a measure of liquidity, we use this Kyle's lambda [6] as a measure of liquidity

[6]The IMFs 2022 liquidity stress-testing framework states directly: Liquidity is also measured as the price impact of trading. Kyles lambda measures the price impact of net trading activities. This is used to diagnose and compare market liquidity across assets and time.

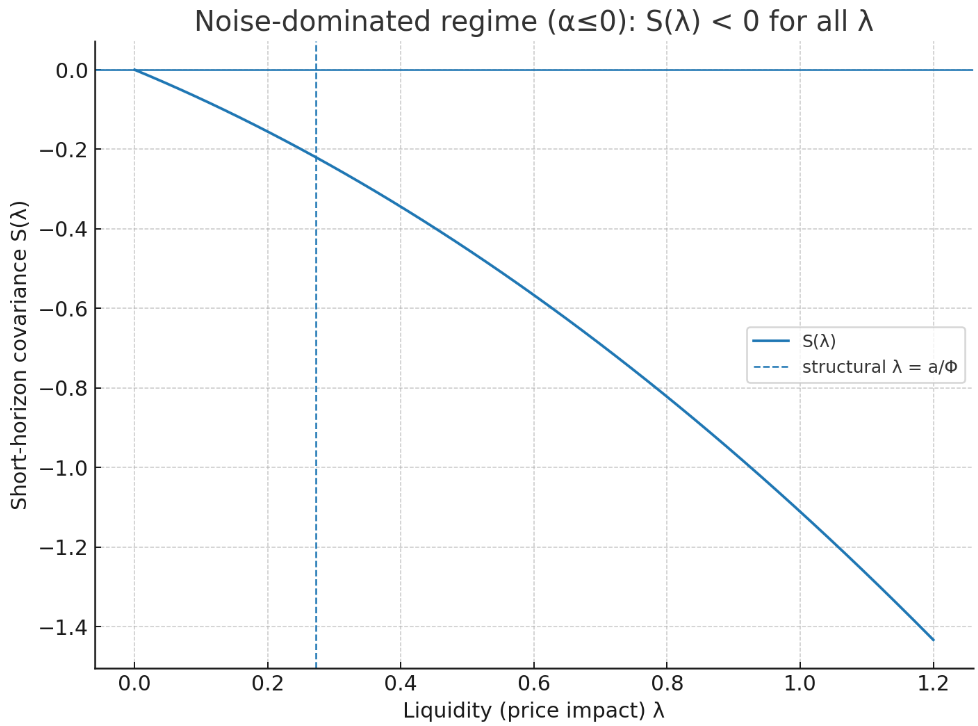

Figure 2: Noisedominated regime ($\alpha_t \leq 0$). $S(\lambda)$ is negative for all $\lambda > 0$, so short-run returns exhibit reversal throughout. The dashed line shows the structural $\lambda = a/\Phi$.

3.1. Short-Horizon Predictability - The "Lambda Law"

Proposition 3.1 (Price and Impact)**.** *Under CARANormal and competitive price taking, the periodt equilibrium price is linear in sufficient statistics*

$$P_t \;=\; \alpha_t\, s_t \;+\; \beta_t\, p_{t^-} \;+\; \lambda_t\, z_t, \qquad \lambda_t \;=\; \frac{a_t}{\Phi_t}, \tag{14}$$

with loadings $\alpha_t = \frac{\mu_t\,\phi_{s,t}}{\Phi_t}$, $\beta_t = \frac{\phi_{p,t^-}}{\Phi_t}$. *Impact* λ_t *is the slope as shown in Kyle(1985) and market depth is defined as* $1/\lambda_t = \Phi_t/a_t$.

Proposition 3.2 (Lambda Law)**.** *The short-horizon autocovariance in the previous subsection at time* t *decomposes as*

$$S_t \;\equiv\; \mathbb{C}\mathrm{ov}(R_t, R_{t+1}) \;=\; \underbrace{\left(\frac{\mu_t(1-\mu_t)(\phi_{s,t}+\phi_{p,t^-})}{a_t^2} - \sigma_{z,t}^2\right)}_{\alpha_t} \lambda_t^2 \;-\; \underbrace{\frac{\mu_t\beta_{t^-}}{a_t}}_{\beta_t} \lambda_t \tag{15}$$

$$\beta_{t^-} \;\equiv\; \frac{\phi_{p,t^-}}{\phi_\theta + \phi_{p,t^-}}. \tag{16}$$

Here λ_t is the price impact as in Kyle(1985) the bigger λ_t, the more price moves when a given amount is traded, which is another way to say the market is thinner and less liquid. The coefficient α_t weighs the strength of learning or how much prices can move toward fundamentals because informed and uninformed traders bring genuine information into the market. this is against the drag from noise trading and inventory pressure, summarized by $\sigma^2_{z,t}$. The coefficient β_t measures a purely mechanical reweighting force - after a public signal arrives, the next trade partially undoes that move as prices lean more on fresher private information, which tends to create short-run reversal. This leads to two important phenomenon which we discuss further.

3.1.1. Noisy Reversals

When $\alpha_t \leq 0$, noise and risk-bearing constraints outweigh the informational pull toward fundamentals. In such a case , $S_t(\lambda_t) < 0$ for every $\lambda_t > 0$. Intuitively, even in already deep markets (small λ_t) any marginal rise in impact makes short-run reversal stronger because the first-order effect of trading is to reweight away from yesterdays public signal; mathematically, $S_t'(0) = -\beta_t < 0$. As liquidity thins further (larger λ_t), the inventory component grows quadratically and pushes S_t more negative, producing the well known snap-back behavior typically seen in stressed markets (see for example Brunnermeier and Pedersen, 2009)

3.1.2. Learning Regime

When $\alpha_t > 0$, learning is strong enough to counter some of the impact drag. Thus, the quadratic opens upward and has a unique point of most reversal at

$$\lambda_{t,\min} = \frac{\beta_t}{2\alpha_t}, \qquad S_t(\lambda_{t,\min}) = -\frac{\beta_t^2}{4\alpha_t} < 0, \tag{17}$$

and a unique sign flip at

$$\lambda_{t,0} = \frac{\beta_t}{\alpha_t}. \tag{18}$$

For impacts below $\lambda_{t,0}$, the reweighting force dominates and short-run returns tend to reverse ($S_t < 0$). Beyond $\lambda_{t,0}$, learning dominates and short-run continuation (in other words, momentum) appears ($S_t > 0$). Economically, getting to the right-hand side usually requires either unusually strong and timely signals (high $\phi_{s,t}$ with a balanced insider share μ_t) or unusually thin markets; However, in many settings the feasible λ_t never reaches that region. The impact that actually prevails is pinned down by depth, $\lambda_t = a_t/\Phi_t$, where $\Phi_t = \phi_\theta + \sum_{k \leq t} \phi_{p,k^-} + \sum_{k \leq t} \mu_k \phi_{s,k}$. The realized regime is therefore determined by where this structural λ_t sits relative to $\lambda_{t,\min}$ and $\lambda_{t,0}$. Policies or technologies that deepen markets (larger Φ_t or smaller a_t) shift λ_t left and make reversal more likely; stronger private information or a more balanced insider share raise α_t and move the thresholds, potentially softening reversal even without changing traded volumes. [7]

4. Insiders Attract Insiders : Dynamic Informed Entry

In the previous section, public signals are already baked into prices, so any advantage from them is mostly competed away(due to market efficiency) which means whats left comes from the market not being perfectly responsive at every moment. Relative to a public signal, a private signal confers two distinct benefits (i) it provides a higher precision estimate of the fundamental value to the observing agent and (ii) it simultaneouly provides a transient oppurtunity to trade before prices fully internalize (see limits to arbitrage). The latter arises as depth is finite and random non-informational order flow keeps prices slightly deviated from their conditional expectation. To add on, with a positive price impact, mean squared pricing errors should be strictly positive meaning an accurate posterior can be profitably deployed. We attempt to endogenize this phenomenon by allowing for dynamic insider participation. Agents become informed when the certainity equivalence between private and public-only forecasts is positive. Hence, insiders are incentivised to enter. Entry raises the mass of informed traders and increases the effective depth (thereby lowered price impact) and tilts price loadings towards private signals thereby paradoxically reducing the certainity equivalence. This, shrinks the information rent. In our model, the value of being informed at date t is precisely the increase in certainty equivalent from replacing a public-only posterior with a public-plus-private posterior while holding the same equilibrium price fixed. We find the certainty equivalent to be quadratic in forecast error and yielding a closed-form.

4.1. Exclusivity Rent of Private Signals

Proposition 4.1 (Certainty Equivalent Gain)**.** *Let $m_{ps,t} \equiv \mathbb{E}[\theta \mid p_{t^-}, s_t]$ and $m_{p,t} \equiv \mathbb{E}[\theta \mid p_{t^-}]$. With the equilibrium pricing rule $P_t = \Psi_t s_t + \Omega_t p_{t^-} + \Lambda_t z_t$, $\Psi_t = \frac{\mu_t \phi_{s,t}}{\Phi_t}$, $\Omega_t = \frac{\phi_{p,t^-}}{\Phi_t}$, $\Lambda_t = \frac{a_t}{\Phi_t}$, the per-capita certainty-equivalent gain from being informed is*

$$\Delta CE_t(\mu_t) = \frac{1}{2A_t}\left(\frac{\mathbb{E}\big[(m_{ps,t} - P_t)^2\big]}{\mathbb{V}\mathrm{ar}(\theta \mid p_{t^-}, s_t)} - \frac{\mathbb{E}\big[(m_{p,t} - P_t)^2\big]}{\mathbb{V}\mathrm{ar}(\theta \mid p_{t^-})}\right). \tag{19}$$

[7] When comparing to data, recall that the feasible impact is typically constrained by the structural value $\lambda_t = a_t/\Phi_t$; the realized regime is determined by the location of λ_t relative to $\lambda_{t,\min}$ and $\lambda_{t,0}$.

The numerators in (19) are mean squared pricing errors under the two information sets, averaged over the unconditional law of $(\theta, \zeta_{t^-}, \tau_t, z_t)$. The denominators rescale by the remaining payoff risk under each information set, reflecting the certainty equivalence. Substituting $\Psi_t, \Omega_t, \Lambda_t$ and the posterior variances $\mathbb{V}\mathrm{ar}(\theta \mid p_{t^-}, s_t) = (\phi_\theta + \phi_{p,t^-} + \phi_{s,t})^{-1}$ and $\mathbb{V}\mathrm{ar}(\theta \mid p_{t^-}) = (\phi_\theta + \phi_{p,t^-})^{-1}$ yields a reduced form in the impact parameter:

$$\Delta CE_t(\mu_t) = \kappa_{1,t}\,\Lambda_t^2 \;-\; \kappa_{2,t}\,\Lambda_t \;+\; \kappa_{0,t}, \tag{20}$$

with the parameters given by

$$\kappa_{1,t} = \frac{\phi_{s,t}}{2\,a_t^3\,\phi_\theta}\Big(a_t^2\phi_\theta\,\sigma_{z,t}^2 + \mu_t^2\phi_{s,t}^2 + \mu_t^2\phi_{s,t}\phi_\theta + 2\mu_t\,\phi_{s,t}\phi_{p,t^-} + \phi_{p,t^-}^2 + \phi_{p,t^-}\phi_\theta\Big) \tag{21}$$

$$\kappa_{2,t} = \frac{\phi_{s,t}}{a_t^2\,\phi_\theta}\Big(\mu_t\,\phi_{s,t} + \mu_t\,\phi_\theta + \phi_{p,t^-}\Big) \tag{22}$$

$$\kappa_{0,t} = \frac{\phi_{s,t}}{2\,a_t\,\phi_\theta} \tag{23}$$

A conveinient way to read the closed form expression is to map each term to its distinct source of informational rent. First, the quadratic component captures the variance of mispricing. This is because in particular, $\kappa_{1,t}$ is linear and scales in the variance of noise trading. In contrast, the linear term in lambda, reflects the predictable repricing away from the public weights. In equilibrium, $\kappa_{1,t}$ and $\kappa_{2,t}$ are strictly positive functions of signals precisions and vanish as depth becomes infinite. Hence, under the regularity conditions of non-degenerate noise trading and strictly positive precisions, the certainity equivalence ΔCE is strictly decreasing on $[0, 1]$. Hence, private information yields the largest rent when few agents are informed and as the entry crowds, the signal into price deepens the market.

4.2. Law of Insider Motion

Classical models of informed trading have long emphasized the impact of insiders on prices and liquidity, but often treated their presence as fixed. For example, Kyle (1985) studied the case of a monopolistic insider facing competitive market makers, while Glosten and Harris (1988) showed how market makers update beliefs and set impact coefficients as a function of order flow. Holden and Subrahmanyam (1992) extended such analysis to multiple insiders, demonstrating that competition among informed traders erodes their informational rents. Yet in all of these frameworks, the number of insiders is taken as exogenous.

Our approach departs from this tradition by endogenizing the population of informed traders. We model insider entry as a dynamic response to profitability. At date t, suppose a fraction μ_t of traders are informed. Each insider earns a certainty-equivalent gain $\Delta CE_t(\mu_t)$. When $\Delta CE_t(\mu_t) > \kappa_t$, where κ_t denotes the cost of acquiring and deploying private information, new entrants are drawn into the market; when $\Delta CE_t(\mu_t) < \kappa_t$, some insiders exit. Instead of assuming a fixed cross-section of beliefs, we endogenize the entry and exit of insiders as a function of informational rents.This generates a simple feedback mechanism

$$\mu_{t+1} \;=\; \Pi_{[0,1]}\Big\{ (1-\delta)\,\mu_t \;+\; \eta\,\Big[\Delta CE_t(\mu_t) - \kappa_t\Big]_+ \Big\}, \qquad \delta \in (0,1],\; \eta > 0, \tag{24}$$

where $[x]_+ = \max\{x, 0\}$ captures onesided entry, δ captures obsolescence/exit, η governs the speed of capacity adjustment, and $\Pi_{[0,1]}$ projects onto $[0,1]$. For settings where smooth interior participation is desirable (e.g., heterogeneous costs or noisy adoption), a smooth alternative that keeps $\mu_{t+1} \in (0,1)$ without truncation is the logit map

$$\mu_{t+1} \;=\; \frac{1}{1+\exp\Big(-\gamma\,[\Delta CE_t(\mu_t) - \kappa_t]\Big)}, \qquad \gamma > 0, \tag{25}$$

which encodes the same economics: profits attract entry; entry deepens markets and competes those profits away by lowering impact $\lambda_t = a_t/\Phi_t$ and tilting prices toward s_t.Figure **??** illustrates how shocks to noise trader variance drive this adjustment process.

In the Kyle (1985) framework, noise variance determines the camouflage available to insiders: when variance is high, insiders can trade more aggressively under the cover of noise, raising their informational rents; when variance is low, their trades become more transparent, compressing rents. In our model, these shocks play the role of perturbations to the insider ecology.

A positive shock to noise variance raises $\Delta CE_t(\mu_t)$ above cost, triggering entry and an upward drift in μ_t. Conversely, a negative shock lowers rents below cost, inducing exit. The “Law of Insider Motion” thus acts as a stabilizing force: shocks displace the insider

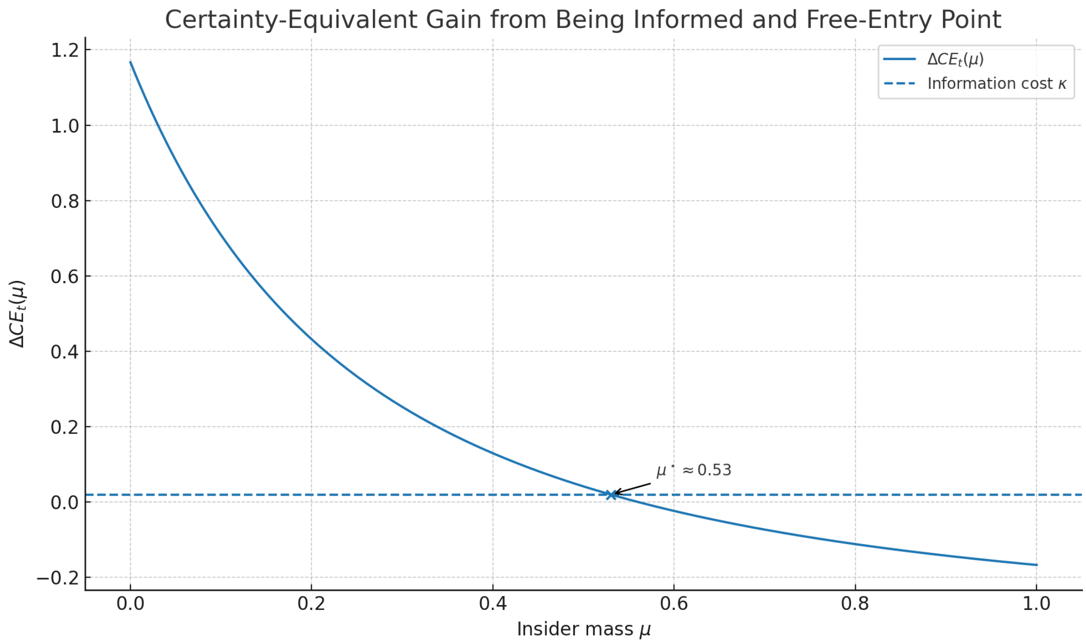

Figure 3: Certaintyequivalent gain from being informed, $\Delta CE_t(\mu)$, as a function of insider mass μ. The solid line shows the decreasing informational rent as participation deepens and price impact falls. The dashed line denotes the fixed information cost κ, and their intersection identifies the unique freeentry equilibrium $\mu^\star$.

population from μ^*, but the feedback mechanism of entry and exit pulls the system back.

4.3. Fixed point and stability

Proposition 4.2 (Monotone value of information and free entry)**.** *Let $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$, $p = \theta + \zeta$ with $\zeta \sim \mathcal{N}(0, \sigma_p^2)$, $s = \theta + \tau$ with $\tau \sim \mathcal{N}(0, \sigma_s^2)$, and $z \sim \mathcal{N}(0, \sigma_z^2)$ mutually independent. Fix date t and suppose the competitive price is*

$$P(\mu) = A(\mu)\, s + B(\mu)\, p + \lambda(\mu)\, z, \qquad A(\mu) = \frac{\mu\, \varphi_s}{\Phi(\mu)}, \quad B(\mu) = \frac{\varphi_p}{\Phi(\mu)}, \quad \lambda(\mu) = \frac{a}{\Phi(\mu)},$$

where $\varphi_\theta = 1/\sigma_\theta^2$, $\varphi_p = 1/\sigma_p^2$, $\varphi_s = 1/\sigma_s^2$ and $\Phi(\mu) = \varphi_\theta + \varphi_p + \mu\varphi_s$. Let $m_{ps} = \mathbb{E}[\theta \mid p, s] = w_p p + w_s s$ with

$$w_p = \frac{\varphi_p}{\varphi_\theta + \varphi_p + \varphi_s}, \qquad w_s = \frac{\varphi_s}{\varphi_\theta + \varphi_p + \varphi_s},$$

and $m_p = \mathbb{E}[\theta \mid p] = \tilde{w}_p p$ with $\tilde{w}_p = \frac{\varphi_p}{\varphi_\theta + \varphi_p}$. Define

$$\Delta CE(\mu) \;=\; \frac{1}{2a}\left(\frac{\mathbb{V}\mathrm{ar}\big(m_{ps} - P(\mu)\big)}{\mathbb{V}\mathrm{ar}(\theta \mid p, s)} - \frac{\mathbb{V}\mathrm{ar}\big(m_p - P(\mu)\big)}{\mathbb{V}\mathrm{ar}(\theta \mid p)}\right),$$

with $\mathbb{V}\mathrm{ar}(\theta \mid p, s) = (\varphi_\theta + \varphi_p + \varphi_s)^{-1}$ and $\mathbb{V}\mathrm{ar}(\theta \mid p) = (\varphi_\theta + \varphi_p)^{-1}$. Then $\Delta CE(\mu)$ is continuous and strictly decreasing on $[0, 1]$. Consequently, for any cost κ the freeentry equation $\Delta CE(\mu^\star) = \kappa$ admits at most one interior solution $\mu^\star \in (0, 1)$ (with corner solutions otherwise).

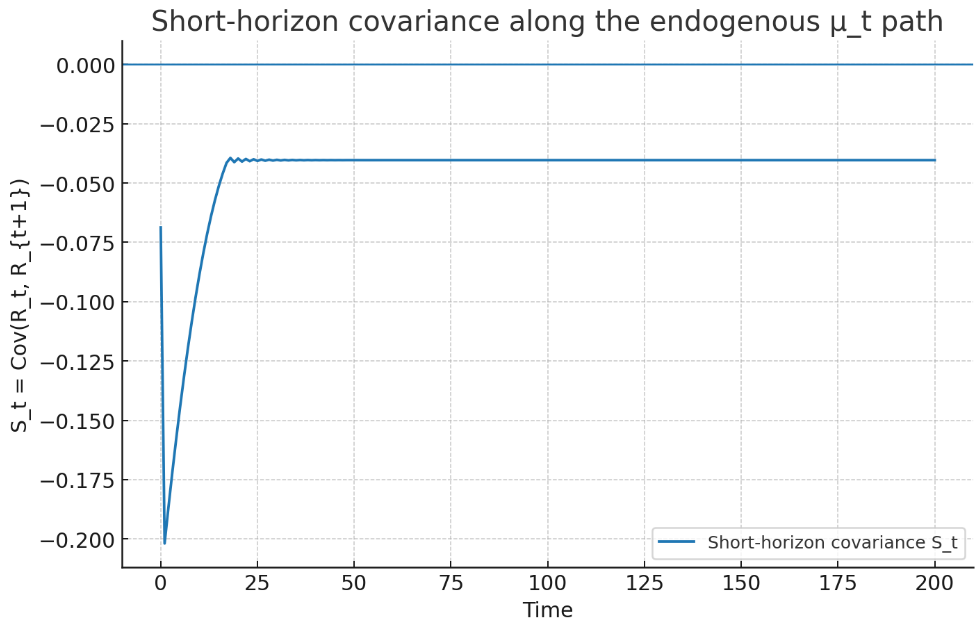

Figure 4: Short-horizon covariance along the equilibrium entry path. As μ_t rises and impact $\lambda_t = a/\Phi_t$ falls, $S_t = \mathbb{C}\mathrm{ov}(R_t, R_{t+1})$ moves along a single-peaked $S(\mu)$ curve and stabilizes at $S(\mu^*)$.

4.4. Free Entry and Participation Dynamics

Insider mass $\mu_t \in [0, 1]$ evolves by free entry according to (24) (or the logit map (25)). Prices are linear in signals with depth $\lambda_t(\mu_t) = a_t/\Phi_t$ and weights that tilt toward the private signal as μ_t rises. The certainty-equivalent gain from becoming informed admits the reduced form

$$\Delta CE_t(\mu) \;=\; \kappa_{1,t}(\mu)\,\lambda_t(\mu)^2 \;-\; \kappa_{2,t}(\mu)\,\lambda_t(\mu) \;+\; \kappa_{0,t}(\mu), \tag{26}$$

with strictly positive coefficients built from signal precisions and noise-trade variance. Proposition **??** establishes that $\Delta CE_t(\mu)$ is strictly decreasing and continuous on $[0, 1]$. As insiders enter, $\Phi_t = \varphi_\theta + \varphi_{p,t-} + \mu_t \varphi_{s,t}$ increases, so $\lambda_t = A_t/\Phi_t$ falls and price loads more heavily on s_t. Both effects compress the mean-squared pricing error that underlies ΔCE_t in (26). Entry thus reduces the private informational rent until (**??**) equilibrium is restored. The application as well as implications of this for short-horizon price covariances are interesting though. From (15), we have

$$\frac{\partial S_t}{\partial \mu} = \underbrace{(2\alpha_t\lambda_t - \beta_t)\lambda_t'}_{\text{impact channel } (\lambda_t'<0)} + \underbrace{\alpha_t'\lambda_t^2 - \beta_t'\lambda_t}_{\text{reweighting toward } s_t}. \tag{27}$$

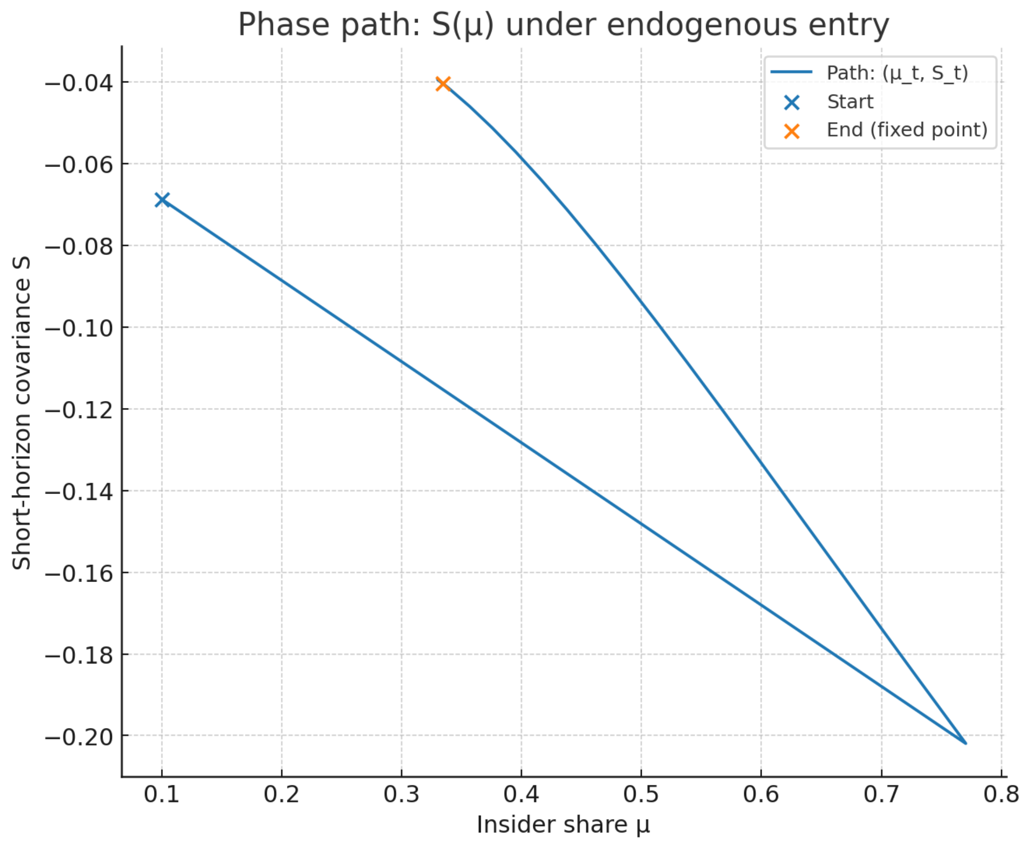

Figure 5: Phase path $S(\mu)$ under endogenous entry. The economy travels from the initial (μ_0, S_0) to the free-entry fixed point $(\mu^*, S(\mu^*))$, illustrating the hump-shaped relation between informed participation and short-run predictability.

When μ is low (i.e. in the case of thin markets), λ_t is large and learning dominates, so $\frac{\partial S_t}{\partial \mu} > 0$. When μ is high, λ_t is small and both reduced impact and public-weight reallocation dominate, giving $\frac{\partial S_t}{\partial \mu} < 0$. Hence $S_t(\mu)$ is unimodal and, in steady state and equals $S_t(\mu_t^\star)$. This has some important implications. First, lower information cost κ_t increases participation $\left(\frac{\partial \mu_t^\star}{\partial \kappa_t} < 0\right)$. Next, higher noise-trade variance $\sigma_{z,t}^2$ raises mispricing (via $\kappa_{1,t}$), so $\frac{\partial \mu_t^\star}{\partial \sigma_{z,t}^2} > 0$. Then higher private precision $\varphi_{s,t}$ raises ΔCE_t directly but deepens the market (reducing λ_t); the net effect on $\mu_t^\star$ and on $S_t(\mu_t^\star)$ is a priori ambiguous. This finally leads to tighter risk-bearing a_t both raises λ_t and scales down the CARA certainty equivalent; the sign of $\frac{\partial \mu_t^\star}{\partial a_t}$ is parameter-dependent. Figure 3 plots $\Delta CE_t(\mu)$ with the free-entry threshold κ and identifies $\mu^\star$.

The phase diagram in §?? traces the implied path of $S_t(\mu)$ and shows its steady-state level $S_t(\mu^\star)$. Figure 7 illustrates how temporary increases in noise-trader variance $(\sigma_{z,t}^2)$ endogenously shift informed participation. A transitory spike in $\sigma_{z,t}^2$ raises the value of private information for a given insider mass μ_t, prompting additional entry so that μ_{t+1} rises. As participation deepens, market depth rebuilds (price impact $\lambda_t = a_t/\Phi_t$ falls), informational rents compress, and μ_t mean-reverts toward the free-entry fixed point μ^*.[8]

4.5. Scenario 1: Shock-Driven Entry Dynamics

To illustrate the momentumcrash ecology under noise shocks, we simulate a discrete-time version of our model with the following parameter values:

Table 1: Parameter values for Scenario 1 simulation

Parameter	**Symbol**	**Value**
Signal precision coefficient	ϕ_θ	1.0
Price adjustment coefficient	ϕ_p	0.5
Inventory sensitivity	ϕ_s	1.0
Market depth scaling	a	1.0
Feedback strength	κ	0.05
Decay rate	η	0.7
Exit/entry friction	δ	0.05
Initial informed participation	μ_0	0.30
Baseline noise variance	σ_z^2	1.0

We impose an exogenous volatility shock at period $t = 10$, which lasts for five periods and multiplies the noise variance $\sigma_{z,t}^2$ by a factor of 4. This generates temporary instability in participation and price impact, followed by endogenous recovery.

Figure 6 reports four panels: (i) the path of noise variance $\sigma_{z,t}^2$, (ii) the participation share μ_t, (iii) the price impact (inverse depth) λ_t, and (iv) the short-horizon return covariance $S_t = \mathrm{Cov}(R_t, R_{t+1})$. The dashed vertical lines mark the beginning and end of the shock episode.

Proposition 4.3. *Let $\mu^\star$ solve the freeentry condition $\Delta CE(\mu^\star; \sigma_z^2) = \kappa$, where*

$$\Delta CE(\mu) = \frac{1}{2a}\left(\frac{\mathbb{V}\mathrm{ar}(m_{ps} - P(\mu))}{\mathbb{V}\mathrm{ar}(\theta \mid p, s)} - \frac{\mathbb{V}\mathrm{ar}(m_p - P(\mu))}{\mathbb{V}\mathrm{ar}(\theta \mid p)}\right),$$

the equilibrium price is linear $P(\mu) = A(\mu)s + B(\mu)p + \lambda(\mu)z$ with

$$A(\mu) = \frac{\mu\phi_s}{\Phi(\mu)}, \qquad B(\mu) = \frac{\phi_p}{\Phi(\mu)}, \qquad \lambda(\mu) = \frac{a}{\Phi(\mu)}, \qquad \Phi(\mu) = \phi_\theta + \phi_p + \mu\phi_s,$$

[8] See the Law of Insider Motion and the figure caption for the entry response and mean reversion.

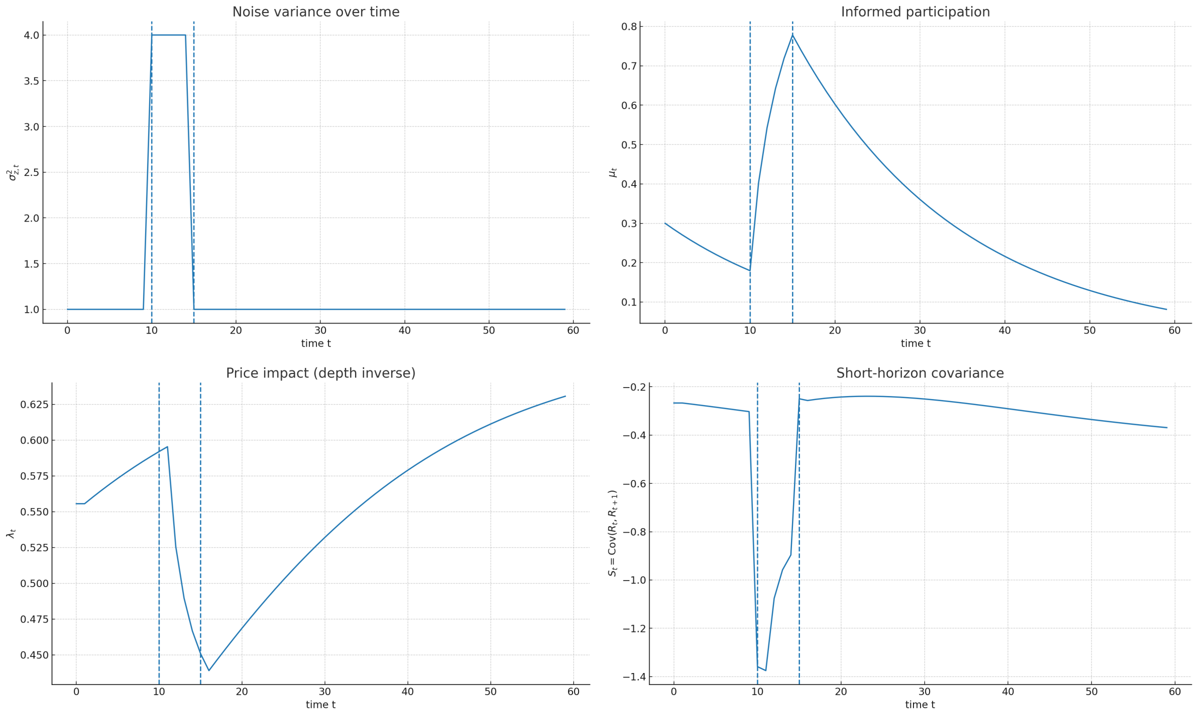

Figure 6: Shock-driven entry dynamics. A volatility shock at $t = 10$ increases $\sigma^2_{z,t}$, triggering a rise in μ_t and transient amplification of price impact λ_t and short-run covariance S_t. The system subsequently reverts toward its stable equilibrium.

and the posterior variances satisfy $\mathbb{V}\text{ar}(\theta \mid p, s) = (\phi_\theta + \phi_p + \phi_s)^{-1}$, $\mathbb{V}\text{ar}(\theta \mid p) = (\phi_\theta + \phi_p)^{-1}$. *Then the partial derivative of the freeentry insider mass with respect to noisetrader variance admits the closed form*

$$\frac{\partial \mu^\star}{\partial \sigma_z^2} = \frac{a^2\, \Phi(\mu^\star)}{2\, \phi_s \left[(\phi_p + \phi_\theta)^2 \sigma_s^2 + \phi_p^2 \sigma_p^2 + a^2 \sigma_z^2\right]} > 0.$$

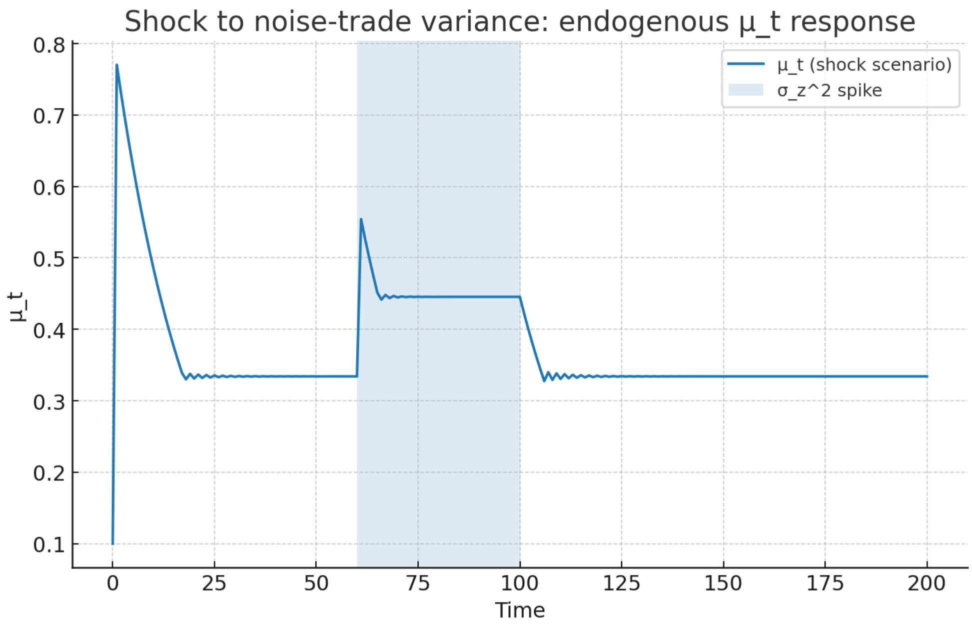

Figure 7: Shock to noise-trader variance and insider entry. A temporary spike in σ_z^2 (shaded) raises the value of information, prompting additional entry and a higher μ_{t+1}; after the shock, μ_t reverts toward μ^* as depth rebuilds.

Appendix A. Backward Induction Results

This appendix derives the linear pricing rule and the adjacent-horizon pricereturn covariances by backward induction under the CARANormal assumptions in the main text. Let the terminal payoff be realized at $t = 5$ so $P_5 = \theta$. Signals are Gaussian and independent across sources and dates: $p_{t^-} = \theta + \zeta_{t^-}$, $s_t = \theta + \tau_t$, $z_t \sim \mathcal{N}(0, \sigma_{z,t}^2)$. We write precisions $\phi_\theta = \mathbb{V}\mathrm{ar}(\theta)^{-1}$, $\phi_{p,t^-} = \mathbb{V}\mathrm{ar}(\zeta_{t^-})^{-1}$, $\phi_{s,t} = \mathbb{V}\mathrm{ar}(\tau_t)^{-1}$, and cumulative precision mass

$$\Phi_t \;\equiv\; \phi_\theta + \sum_{k\le t} \phi_{p,k^-} + \sum_{k\le t} \mu_k\, \phi_{s,k}. \tag{A.1}$$

while defining the public weight $\beta_{t^-} \equiv \phi_{p,t^-}/(\phi_\theta + \phi_{p,t^-})$ and (when used) the posterior variances $\mathbb{V}\mathrm{ar}(\theta \mid p_{t^-}) = (\phi_\theta + \phi_{p,t^-})^{-1}$, $\mathbb{V}\mathrm{ar}(\theta \mid p_{t^-}, s_t) = (\phi_\theta + \phi_{p,t^-} + \phi_{s,t})^{-1}$.

Starting on the backward induction process, at $t = 4$ agents trade one last time before $P_5 = \theta$. CARANormal and competitive price taking imply myopic demands for an agent with information $\mathcal{H} \in \{\mathcal{F}_{4^-}, \mathcal{I}_4\}$,

$$X_4(\mathcal{H}) \;=\; \frac{\mathbb{E}[\theta \mid \mathcal{H}] - P_4}{a_4\, \mathbb{V}\mathrm{ar}(\theta \mid \mathcal{H})}.$$

Here $\mathcal{F}_{4^-} = \sigma(p_{4^-})$ which is public information and $\mathcal{I}_4 = \sigma(p_{4^-}, s_4)$ which is insider

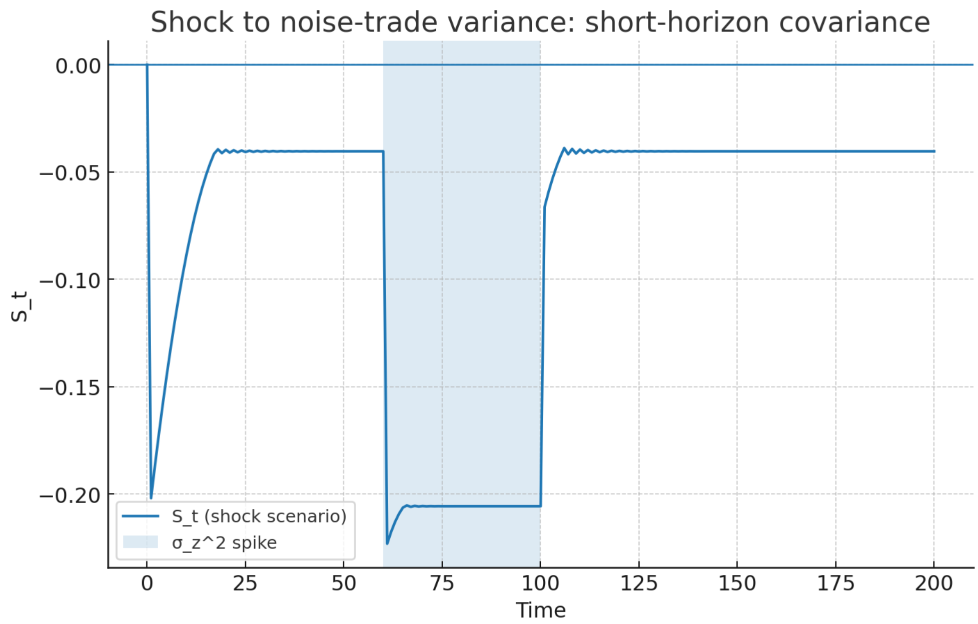

Figure 8: Shock to noise-trader variance and short-horizon covariance. During the σ_z^2 spike (shaded) inventory pressure dominates and S_t becomes more negative (stronger reversal). As entry deepens markets post-shock, S_t relaxes back toward $S(\mu^*)$.

information. With a fraction μ_4 informed, market clearing gives

$$\mu_4 \frac{\mathbb{E}[\theta \mid p_{4-}, s_4] - P_4}{a_4 \, \mathbb{V}\mathrm{ar}(\theta \mid p_{4-}, s_4)} \;+\; (1-\mu_4) \frac{\mathbb{E}[\theta \mid p_{4-}] - P_4}{a_4 \, \mathbb{V}\mathrm{ar}(\theta \mid p_{4-})} \;+\; z_4 \;=\; 0. \tag{A.2}$$

Solving for P_4 and inserting conjugate Normal posteriors yields the linear rule

$$P_4 \;=\; \alpha_4 \, s_4 + \beta_4 \, p_{4-} + \lambda_4 \, z_4, \qquad A_4 = \frac{\mu_4 \, \phi_{s,4}}{\Phi_4}, \quad B_4 = \frac{\phi_{p,4-}}{\Phi_4}, \quad \lambda_4 = \frac{a_4}{\Phi_4}. \tag{A.3}$$

Let wealth evolve as $W_{t+1} = W_t + x_t \, (P_{t+1} - P_t)$, and define one-step return $R_{t+1} \equiv P_{t+1} - P_t$. Write $\mathcal{F}_t$ for the agent's information set at t.

Lemma Appendix A.1 (Stagewise optimality (myopia))**.** *Under Assumption 2.1, the Bellman problem at time t reduces to a one-period meanvariance trade:*

$$x_t^\star \;=\; \arg\max_{x_t} \left\{ x_t \, m_t \;-\; \tfrac{a_t}{2} \, v_t \, x_t^2 \right\} \;=\; \frac{m_t}{a_t \, v_t}, \qquad m_t \equiv \mathbb{E}_t[R_{t+1}], \quad v_t \equiv \mathbb{V}\mathrm{ar}_t(R_{t+1}).$$

In particular, with terminal payoff $P_5 = \theta$ and linear pricing in the main text, $m_t = \mathbb{E}_t[\theta] - P_t$ and $v_t = \mathbb{V}\mathrm{ar}_t(\theta)$, yielding

$$x_t^\star \;=\; \frac{\mathbb{E}_t[\theta] - P_t}{a_t \, \mathbb{V}\mathrm{ar}_t(\theta)}.$$

Proof of Proposition. Consider the Bellman equation with continuation value summarized by the future optimized gains $H_{t+1} \equiv \sum_{u=t+1}^{4} x_u^\star R_{u+1}$:

$$V_t(W_t, \mathcal{F}_t) = \max_{x_t} \; \mathbb{E}_t\Big[-\exp\Big\{-a_t\Big(W_t + x_t R_{t+1} + H_{t+1}\Big)\Big\}\Big].$$

(CARA additivity.) Factor out wealth:

$$V_t(W_t, \mathcal{F}_t) = -e^{-a_t W_t} \min_{x_t} \; \mathbb{E}_t\Big[\exp\Big\{-a_t\Big(x_t R_{t+1} + H_{t+1}\Big)\Big\}\Big].$$

(Exogenous opportunities.) Under price taking and exogenous information, H_{t+1} is independent of x_t conditional on $\mathcal{F}_t$. Hence the x_t-problem is

$$\min_{x_t} \; \mathbb{E}_t\Big[e^{-a_t x_t R_{t+1}}\Big].$$

(Gaussian certainty equivalence.) Since $R_{t+1} \mid \mathcal{F}_t \sim \mathcal{N}(m_t, v_t)$, its mgf gives

$$\mathbb{E}_t\Big[e^{-a_t x_t R_{t+1}}\Big] = \exp\Big\{-a_t x_t m_t + \tfrac{1}{2} a_t^2 x_t^2 v_t\Big\}.$$

Maximizing expected utility is thus equivalent to maximizing the certainty equivalent[9] $x_t m_t - \frac{a_t}{2} v_t x_t^2$, with FOC $m_t - a_t v_t x_t = 0$ and SOC $-a_t v_t < 0$, yielding $x_t^\star = m_t/(a_t v_t)$. Under the papers payoff structure, $m_t = \mathbb{E}_t[\theta] - P_t$ and $v_t = \mathbb{V}\mathrm{ar}_t(\theta)$, completing the proof. □

Proof of Proposition 3.1. Assume for some $t \in \{2, 3, 4\}$ that the equilibrium price at date t is linear as in (A.3) with $\Psi_t = \mu_t \phi_{s,t}/\Phi_t$, $\Omega_t = \phi_{p,t^-}/\Phi_t$, $\Lambda_t = a_t/\Phi_t$. We show it implies the same structure at $t-1$ and delivers the adjacent-horizon covariance. At $t-1$, informed and uninformed demands are, respectively,

$$X_{I,t-1} = \frac{\mathbb{E}[\theta \mid p_{(t-1)^-}, s_{t-1}] - P_{t-1}}{a_{t-1}\, \mathbb{V}\mathrm{ar}(\theta \mid p_{(t-1)^-}, s_{t-1})}, \tag{A.4}$$

$$\tag{A.5}$$

$$X_{U,t-1} = \frac{\mathbb{E}[\theta \mid p_{(t-1)^-}] - P_{t-1}}{a_{t-1}\, \mathbb{V}\mathrm{ar}(\theta \mid p_{(t-1)^-})}. \tag{A.6}$$

Market clearing is given by

$$\mu_{t-1} X_{I,t-1} + (1 - \mu_{t-1}) X_{U,t-1} + z_{t-1} = 0 \tag{A.7}$$

[9] Intertemporal hedging demands arise when current positions covary with future changes in the opportunity set. Here, the state that governs opportunities is the belief about θ, which evolves only via exogenous future signals; with CARA and Normality, the value function is exponentialaffine in wealth and introduces no cross terms that couple x_t to the law of future states. Myopia can fail if (a) trades have price impact that feeds back into future coefficients, (b) the information flow is endogenous to order flow, (c) utility is non-CARA or shocks are non-Gaussian, or (d) there are priced state variables (time-varying drifts/vols) that are correlated with returns and affected by x_t.

produces

$$P_{t-1} \;=\; \Psi_{t-1}\, s_{t-1} + \Omega_{t-1}\, p_{(t-1)^-} + \Lambda_{t-1}\, z_{t-1} \tag{A.8}$$

$$\tag{A.9}$$

$$\Psi_{t-1} = \frac{\mu_{t-1}\phi_{s,t-1}}{\Phi_{t-1}}, \quad \Omega_{t-1} = \frac{\phi_{p,(t-1)^-}}{\Phi_{t-1}}, \quad \Lambda_{t-1} = \frac{a_{t-1}}{\Phi_{t-1}}, \tag{A.10}$$

with

$$\Phi_{t-1} = \phi_\theta + \sum_{k\le t-1} \phi_{p,k^-} + \sum_{k\le t-1} \mu_k \phi_{s,k} \tag{A.11}$$

Thus the linear form and the depth/impact identity $\Lambda_{t-1} = a_{t-1}/\Phi_{t-1}$ propagate backwards. Write the price at any $u \le 4$ as $P_u = (\Psi_u+\Omega_u)\theta + \Psi_u\tau_u + \Omega_u\zeta_{u^-} + \Lambda_u z_u$, using $s_u = \theta+\tau_u$, $p_{u^-} = \theta + \zeta_{u^-}$. We define one-step returns $R_u \equiv P_{u+1} - P_u$. For the *last interior step* $t = 3$ (so that $R_4 = P_5 - P_4 = \theta - P_4$),

$$\begin{aligned} R_3 \;&=\; \Big[(\Psi_4{+}\Omega_4) - (\Psi_3{+}\Omega_3)\Big]\theta \;+\; \Psi_4\tau_4 + \Omega_4\zeta_{4^-} + \Lambda_4 z_4 \;-\; \Psi_3\tau_3 - \Omega_3\zeta_{3^-} - \Lambda_3 z_3, \\ R_4 \;&=\; \Big[1 - (\Psi_4{+}\Omega_4)\Big]\theta \;-\; \Psi_4\tau_4 \;-\; \Omega_4\zeta_{4^-} \;-\; \Lambda_4 z_4. \end{aligned}$$

Independence across dates and shocks implies only terms sharing the same innovation survive in $\mathbb{C}\mathrm{ov}(R_3, R_4)$ given as

$$\mathbb{C}\mathrm{ov}(R_3,R_4) = \underbrace{\Big[(\Psi_4{+}\Omega_4) - (\Psi_3{+}\Omega_3)\Big]\Big[1-(\Psi_4{+}\Omega_4)\Big]\mathbb{V}\mathrm{ar}(\theta)}_{\text{fundamental block}} \;-\; \underbrace{\Psi_4^2\,\mathbb{V}\mathrm{ar}(\tau_4) + \Omega_4^2\,\mathbb{V}\mathrm{ar}(\zeta_{4^-}) + \Lambda_4^2\,\sigma_{z,4}^2}_{\text{price-pressure (inventory)}} \tag{A.12}$$

An identical calculation gives, for a generic interior pair $(t, t+1)$ gives us,

$$\mathbb{C}\mathrm{ov}(R_t,R_{t+1}) = \Big[(\Psi_{t+1}{+}\Omega_{t+1}){-}(\Psi_t{+}\Omega_t)\Big]\Big[(\Psi_{t+2}{+}\Omega_{t+2}){-}(\Psi_{t+1}{+}\Omega_{t+1})\Big]\mathbb{V}\mathrm{ar}(\theta){-}\Psi_{t+1}^2\mathbb{V}\mathrm{ar}(\tau_{t+1}){-}\Omega_{t+1}^2\mathbb{V}\mathrm{ar} \tag{A.13}$$

Obtaining closed form expressions for covariances yields the compact quadratic in impact[10]

$$\mathbb{C}\mathrm{ov}(R_3,R_4) \;=\; \underbrace{\left(\frac{\mu_3(1-\mu_3)(\phi_{s,3}+\phi_{p,3^-})}{a_3^2} - \sigma_{z,3}^2\right)}_{\alpha_3}\Lambda_3^2 \;-\; \underbrace{\frac{\mu_3\,\Omega_{3^-}}{a_3}}_{\beta_3}\Lambda_3, \tag{A.14}$$

with $\Omega_{3^-} = \frac{\phi_{p,3^-}}{\phi_\theta+\phi_{p,3^-}}$, which is the *Lambda Law* stated in the paper for the last interior step. □

[10] For a non-terminal interior pair $(t, t+1)$, (A.13) is the primitive expression. When $t+2$ is close to terminal or receives only payoff news, the first term simplifies exactly as above and collapses to the quadratic in Λ_t.

Proof of 4.2. By independence of (p, s, z) and linearity,

$$\mathbb{V}\mathrm{ar}(m_{ps} - P) = (w_s - A)^2\sigma_s^2 + (w_p - B)^2\sigma_p^2 + \lambda^2\sigma_z^2. \tag{A.15}$$
$$\mathbb{V}\mathrm{ar}(m_p - P) = (\tilde{w}_p - B)^2\sigma_p^2 + A^2\sigma_s^2 + \lambda^2\sigma_z^2. \tag{A.16}$$

With $\Phi(\mu) = \varphi_\theta + \varphi_p + \mu\varphi_s$, compute

$$\begin{aligned}
\Phi'(\mu) &= \varphi_s \\
\lambda'(\mu) &= -\frac{a\,\varphi_s}{\Phi(\mu)^2} \\
A'(\mu) &= \frac{\varphi_s(\varphi_\theta + \varphi_p)}{\Phi(\mu)^2} \\
B'(\mu) &= -\frac{\varphi_p\,\varphi_s}{\Phi(\mu)^2}
\end{aligned}$$

where $\Phi(\mu) = \varphi_\theta + \varphi_p + \mu\varphi_s$. and $\Phi(\mu) = \varphi_\theta + \varphi_p + \mu\varphi_s$ with w_p, w_s, and $\tilde{w}_p$ are constants. With

$$\mathbb{V}\mathrm{ar}(m_{ps} - P) = (w_s - A)^2\sigma_s^2 + (w_p - B)^2\sigma_p^2 + \lambda^2\sigma_z^2 \tag{A.17}$$
$$\mathbb{V}\mathrm{ar}(m_p - P) = (\tilde{w}_p - B)^2\sigma_p^2 + A^2\sigma_s^2 + \lambda^2\sigma_z^2, \tag{A.18}$$

we obtain

$$\frac{\partial}{\partial\mu}\,\mathbb{V}\mathrm{ar}(m_{ps} - P) = -\frac{2\varphi_s}{\Phi(\mu)^2}\Big[(\varphi_\theta + \varphi_p)(w_s - A)\sigma_s^2 + \varphi_p(B - w_p)\sigma_p^2\Big] \;-\; \frac{2a^2\varphi_s}{\Phi(\mu)^3}\,\sigma_z^2 \tag{A.19}$$

$$\frac{\partial}{\partial\mu}\,\mathbb{V}\mathrm{ar}(m_p - P) = \frac{2\varphi_s}{\Phi(\mu)^2}\Big[\varphi_p(\tilde{w}_p - B)\sigma_p^2 + (\varphi_\theta + \varphi_p)A\,\sigma_s^2\Big] \;-\; \frac{2a^2\varphi_s}{\Phi(\mu)^3}\,\sigma_z^2. \tag{A.20}$$

where the first derivative is strictly negative on $[0, 1)$, while the second is *a priori* signindeterminate. Since the posterior variances are constants,

$$\frac{\partial\Delta CE(\mu)}{\partial\mu} = \frac{\varphi_s^2}{\Phi(\mu)^3\,a}\Big\{\sigma_s^2\,(\mu\varphi_s - \Phi(\mu))\,(\varphi_p + \varphi_\theta) \;-\; \varphi_p^2\,\sigma_p^2 \;-\; a^2\,\sigma_z^2\Big\} \tag{A.21}$$

$$= -\frac{\varphi_s^2}{a\Big(\varphi_\theta + \varphi_p + \mu\varphi_s\Big)^3}\Big\{(\varphi_p + \varphi_\theta)^2\,\sigma_s^2 \;+\; \varphi_p^2\,\sigma_p^2 \;+\; a^2\,\sigma_z^2\Big\} \;<\; 0. \tag{A.22}$$

Thus $\frac{\partial\Delta CE(\mu)}{\partial\mu} < 0$ for all $\mu \in [0, 1)$, with continuity at $\mu = 1$ by continuity of loadings. This shows strict monotonicity and continuity. Since $\Delta CE(\mu)$ is strictly decreasing and continuous on $[0, 1]$, the scalar equation $\Delta CE(\mu^\star) = \kappa$ has at most one interior solution. Existence of an interior solution is ensured whenever $\Delta CE(0) \geq \kappa \geq \Delta CE(1)$; otherwise a corner $\mu^\star \in \{0, 1\}$ obtains. □

Proof. By linearGaussian pricing, the two meansquared pricing errors contain the common noisepressure term $\lambda(\mu)^2\sigma_z^2$:

$$\mathbb{V}\mathrm{ar}(m_{ps} - P) = \cdots + \lambda^2\sigma_z^2, \qquad \mathbb{V}\mathrm{ar}(m_p - P) = \cdots + \lambda^2\sigma_z^2.$$

Hence $\partial \Delta CE / \partial \sigma_z^2$ (with fixed μ) is

$$\frac{\partial \Delta CE}{\partial \sigma_z^2} = \frac{1}{2a}\left(\frac{\lambda^2}{\mathbb{V}\mathrm{ar}(\theta \mid p, s)} - \frac{\lambda^2}{\mathbb{V}\mathrm{ar}(\theta \mid p)}\right) \tag{A.23}$$

$$= \frac{\lambda^2}{2a}\Big[(\phi_\theta + \phi_p + \phi_s) - (\phi_\theta + \phi_p)\Big] \tag{A.24}$$

$$= \frac{\lambda^2}{2a}\phi_s \tag{A.25}$$

$$= \frac{a^2}{\Phi(\mu)^2} \cdot \frac{\phi_s}{2a} \tag{A.26}$$

$$= \frac{a\,\phi_s}{2\,\Phi(\mu)^2}. \tag{A.27}$$

(ii) *Slope in μ.* Appendix A yields

$$\frac{\partial \Delta CE}{\partial \mu} = -\frac{\phi_s^2}{a}\,\frac{1}{\Phi(\mu)^3}\Big\{(\phi_p + \phi_\theta)^2\sigma_s^2 + \phi_p^2\sigma_p^2 + a^2\sigma_z^2\Big\} < 0 \quad \text{for all } \mu \in [0, 1).$$

(iii) *Implicit Function Theorem.* Differentiating the freeentry condition $\Delta CE(\mu^\star, \sigma_z^2) = \kappa$ w.r.t. σ_z^2 (holding κ fixed) gives

$$\frac{\partial \Delta CE}{\partial \mu}(\mu^\star)\,\frac{\partial \mu^\star}{\partial \sigma_z^2} + \frac{\partial \Delta CE}{\partial \sigma_z^2}(\mu^\star) = 0 \quad \Rightarrow \quad \frac{\partial \mu^\star}{\partial \sigma_z^2} = -\frac{\dfrac{\partial \Delta CE}{\partial \sigma_z^2}(\mu^\star)}{\dfrac{\partial \Delta CE}{\partial \mu}(\mu^\star)}.$$

Substituting the expressions from (i)(ii) and simplifying:

$$\frac{\partial \mu^\star}{\partial \sigma_z^2} = \frac{\dfrac{a\,\phi_s}{2\,\Phi(\mu^\star)^2}}{\dfrac{\phi_s^2}{a}\,\dfrac{1}{\Phi(\mu^\star)^3}\Big\{(\phi_p + \phi_\theta)^2\sigma_s^2 + \phi_p^2\sigma_p^2 + a^2\sigma_z^2\Big\}} = \frac{a^2\,\Phi(\mu^\star)}{2\,\phi_s\,\Big[(\phi_p + \phi_\theta)^2\sigma_s^2 + \phi_p^2\sigma_p^2 + a^2\sigma_z^2\Big]}.$$

Positivity follows because the numerator and the bracketed denominator term are positive.

□ □